



Global Credit Data

by banks for banks

**A Theoretical and Empirical Analysis of Alternative Discount
Rate Concepts for Computing LGDs using Historical Bank
Workout Data**

- **Proceedings from the GCD Discount Rate Working Group**

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Table of Contents

1	Executive summary	3
2	LGD terminology	4
3	Risk, expectation and realization	5
3.1	Default risk	5
3.2	Resolution risk.....	6
3.3	Estimation of LGD	7
4	Discount rates for computing empirical LGDs	7
4.1	Regulatory guidance	7
4.2	Guiding principles for LGD discount rates	9
4.3	Review of discount rate approaches.....	10
4.3.1	Loan contract rate.....	10
4.3.2	Bank weighted average cost of capital (WACC).....	11
4.3.3	Bank cost of equity.....	13
4.3.4	Market return of marketable credit instrument.....	14
4.3.5	Market equilibrium return	14
4.4	Benchmarking of discount rate approaches	16
5	Applying different discount rates to GCD workout data	19
5.1	Using GCD workout data for empirical discount rate studies.....	19
5.1.1	Screening the data and applying data filters	19
5.1.2	Risk segmentation.....	20
5.1.3	Empirical strategy	23
5.2	Empirical discount rate analysis.....	24
5.2.1	Assumptions.....	24
5.2.2	Euro risk free rates.....	27
5.2.3	Currency-specific risk free rates.....	29
5.3	Empirical LGD analysis	31
5.3.1	Euro risk free rates.....	31
5.3.2	Correction for time to resolution bias	32
5.3.3	Currency-specific risk free rates.....	36
6	Outlook	38
7	References	39

1 Executive summary

This paper summarizes the discussions of the Global Credit Data (GCD) Discount Rate Work Group (WG). The WG does not provide a “correct” number for the discount rate but derives guiding principles for Loss rate Given Default (LGD) discount rate approaches.

The WG has analyzed five main discount rate approaches:

- Contract rate at origination (Contract): contractual interest rate of a loan, which is the combination of a base rate plus a risk premium at loan origination;
- Loan weighted average cost of capital (WACC): bank funding costs, which is the combination of the capital ratio weighted cost of equity funding and the debt ratio weighted cost of debt funding. The capital ratio may be based on the loan-specific regulatory capital requirement;
- Return on equity (ROE): cost of equity funding;
- Market return on defaulted debt: return on the bond price at resolution (i.e., the realized recovery at resolution) relative to the bond price at default (i.e., the expected recovery at default);
- Market equilibrium return: based on the combination of a base rate plus a premium for systematic risk. The risk premium is based on the product of the systematic risk sensitivity and the equity risk premium.

The key theoretical findings of the WG are:

- Appropriate discount rates are based on a combination of the risk free rate and risk premium for systematic risk at the time of default;
- Discount rates should not compensate for realized risk;
- Approaches may be separated into contract specific, comparable and equilibrium approaches;
- WACC and market equilibrium return have a slight preference in the Working Group;
- Contract rates are popular in practice as they have practical advantages: they adjust automatically for changing supply/demand and risk aversion in the economy and align with accounting requirements. However, contract rates have shortcomings:
 - Rates are based on the origination time: this is a violation of the principle that discount rates should relate to the default time;
 - Rates include the expected loss in relation to default risk: this is a violation of the principle that discount rates should not compensate for realized risk.

The key empirical findings of the WG are:

- Declining risk free rates are in part offset by increasing market risk premiums in the period 2000-2013;
- Common empirical discount rates are found to be between the risk free rate and the return on equity;
- The variation of empirical LGDs is moderate for the various discount rate approaches;
- GCD may consider providing simple measures for systematic risk for homogenous risk segments to facilitate the appropriate reflection of systematic risk in the discount rates. Risk measures may include asset correlations and variation coefficients.

2 LGD terminology

The Loss rate Given Default (LGD) is a central modeling parameter in bank-internal credit risk models for credit risk exposures and measures the degree of shortfall of net recoveries relative to the outstanding loan amount **due to and attributable to the default event**.¹ Credit risk exposures may result from assets, derivatives, credit lines and guarantees.

GCD has established a terminology of “probability of default” being the forward estimate and “observed default frequency” or “observed default rate” being the observed. In this spirit, the WG has established a clear and simple terminology for LGDs, which is summarized in Table 1:

Table 1: LGD Terminology

Abbrev.	Description	Character	Discount	Application
LGD	LGD	Risk (random variable)	yes	Model framework
NLGD	Nominal LGD	Risk (random variable)	no	Model framework
ELGD	Expected LGD	Expectation	yes	Stage II LGD modeling
ENLGD	Expected Nominal LGD	Expectation	no	Avoid circular references
OLGD	Observed LGD	Realization	yes	Stage I LGD modeling
ONLGD	Observed Nominal LGD	Realization	no	Avoid circular references
DLGD ²	Basel downturn LGD	Stress	n.a.	Regulatory capital

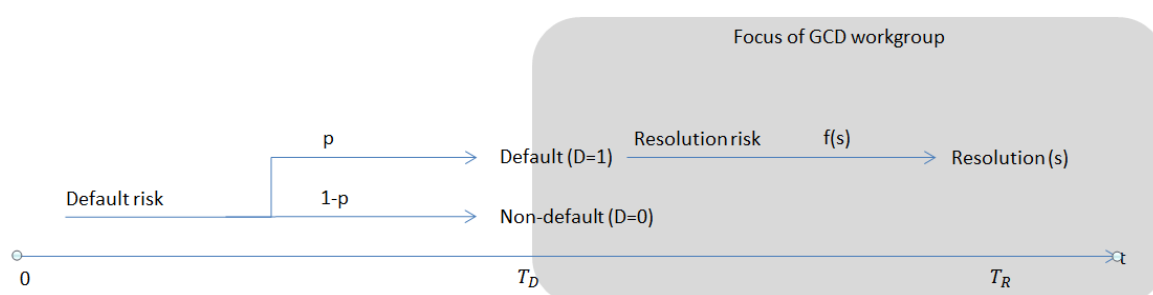
¹ Net recoveries are gross recoveries less costs associated with the recovery process.

² Downturn LGD is subject to regulatory definition and is not the focus of this study.

3 Risk, expectation and realization

Figure 1 shows the stylized evolution of credit losses from the beginning of the observation period (time 0) to the default event (time T_D), and to the resolution event (time T_R).³ We consider two sequential risk processes: **default risk** with two possible outcomes (default and non-default), and **resolution risk** with an infinite number of possible outcome states S . These outcomes may include a random number of cash flows at random times between default time and resolution time. Examples for resolution categories are workouts, modifications, and cures.⁴

Figure 1: Evolution of credit losses



3.1 Default risk

Default risk is assumed to have two possible realizations $D \in \{0,1\}$. The associated random loss rate given the random LGD at time of default is:

$$L_{T_D} = \begin{cases} 0, & \text{if } D=0 \\ LGD, & \text{if } D=1 \end{cases} \quad (1)$$

The variable LGD will be considered in detail in the next section. The probability distribution of the default indicator D is Bernoulli:

$$p(D) = \begin{cases} 1 - p, & \text{if } D=0 \\ p, & \text{if } D=1 \end{cases} \quad (2)$$

The outcome of default risk is random (hence unknown) and the expected loss rate (EL) at default may be computed by summing up the probability weighted discrete loss outcomes:

$$EL_{T_D} = (1 - p) * 0 + p * E(LGD_{T_D}) = p * E(LGD_{T_D}) \quad (3)$$

The present value at origination of future loan losses results from discounting the outcomes to the origination (time T_0):

³ Some reference values for the average time to resolution may be found in Araten et al. (2004): 0.8 years and Jacobs Jr (2012): 1.7 years. These values are consistent with the GCD experience.

⁴ GCD has collected a large number of observations of default outcomes (resolutions).

$$EL_0 = (1 - p) * 0 + p * E \left(\frac{LGD_{T_D}}{(1 + rf_{[0, T_D]} + \delta_{[0, T_D]})^{T_D - 0}} \right) = \frac{p * E(LGD_{T_D})}{(1 + rf_{[0, T_D]} + \delta_{[0, T_D]})^{T_D - 0}}. \quad (4)$$

The discount time and discount rate is equal to the risk free rate $rf_{[0, T_D]}$ and a risk premium $\delta_{[0, T_D]}$ for all resolution outcomes and can hence be written outside the expectation operator as all resolution outcomes relate to the same time of default.

The discount rate for default risk includes a risk premium that is based on the systematic risk of the default process and is (state) independent of the (unknown) outcome of default risk as all risk premiums are (compare Damodaran (2007) and the guiding principles for risk based discount rates in Section 4.2). The discount rate may be period-specific ($[0, T_D]$) to reflect the term structure of discount rates.

3.2 Resolution risk

Resolution risk is assumed to have an infinite amount of possible realizations. The random net recovery cash flows for a given realization path s may occur any time between default time and the resolution time: $c(s) = (c_{t_1}(s), \dots, c_{T_R}(s))$ and the probability density function is $f(s)$, with $\int f(s) = 1$.

The random absolute (i.e., in dollar terms) nominal LGD (ANLGD) is then:

$$ANLGD(s) = EAD - \sum_{t=T_D}^{T_R} c_t(s). \quad (5)$$

The random relative nominal LGD (NLGD) per scenario is computed by relating ANLGD to the exposure at default (EAD), which comprises the outstanding principal and accrued interest and is assumed to be deterministic, rather than an ex-ante bank estimate:⁵

$$NLGD(s) = \frac{ANLGD(s)}{EAD} = 1 - EAD^{-1} \sum_{t=T_D}^{T_R} c_t(s). \quad (6)$$

The random LGD at default time follows by the means of discounting by a risk-adjusted rate:

$$LGD(s) = 1 - EAD^{-1} \sum_{t=t_1(s)}^{T_R(s)} \frac{c_t(s)}{(1 + rf_{[T_D, t]} + \delta_{[T_D, t]})^{t - T_D}}. \quad (7)$$

The discount rate is the same for all loss observations and (state) independent of the (unknown) outcome of resolution risk as all risk premiums are (compare the guiding principles for risk based

⁵ Corporate loans often have deterministic amortisation schedules rather than embedded pre-payment options and liquidity facilities.

discount rates in Section 4.2). The objective of the Working Group is the computation of observed LGDs based on the time value of money – here $r f_{[T_D, t]} + \delta_{[T_D, t]}$. The discount rate may be period-specific ($[T_D, t]$) to reflect the term structure of discount rates.

The outcome of resolution risk is random (hence unknown ex-ante) and the expected nominal LGD (ENLGD), which is unconditional on any risk factor, may be computed by integrating over the density weighted random $NLGD(s)$:

$$ENLGD = E(NLGD) = \int f(s)NLGD(s) ds. \quad (8)$$

Likewise, the present value of the expected LGD is:

$$ELGD = E(LGD) = \int f(s)LGD(s) ds. \quad (9)$$

3.3 Estimation of LGD

The observations of the random variables NLGD (LGD) may be called observed NLGD (LGD), realized NLGD (LGD), ultimate NLGD (LGD), or post resolution NLGD (LGD). We will call this observed nominal LGD (ONLGD) and observed LGD (OLGD), which can be estimated, based on Equations (6) and (7). This is often referred to as Stage I LGD modelling.

The observed LGDs may be linked with risk factors (via the estimation of regression models) and LGD may be estimated conditional on risk factors. The resulting estimated LGDs may be applied to compute bank-internal and regulatory (Basel II/III) capital. This is often referred to as Stage II LGD modelling.

Furthermore, the expected NLGD (LGD) may be estimated based on assumptions. One example for this estimation is the average over the ONLGDs (OLGDs) for given risk segments.

4 Discount rates for computing empirical LGDs

4.1 Regulatory guidance

The Basel Committee on Banking Supervision (2005a and 2005b) mandates that the discount rate includes the time value of money and a risk premium for undiversifiable risk:

“[...] When recovery streams are uncertain and involve risk that cannot be diversified away, net present value calculations must reflect the time value of money and a risk premium appropriate to the **undiversifiable risk**. In establishing appropriate risk premiums for the estimation of LGDs consistent with economic downturn conditions, the bank should focus on the uncertainties in recovery cash flows associated with defaults that arise during the economic downturn conditions identified

under Principle 1. When there is no uncertainty in recovery streams (e.g., recoveries derived from cash collateral), net present value calculations need only reflect the time value of money, and a risk free discount rate is appropriate.”

Variations of these regulations and guidance notes are included in national guidance notes from prudential regulators. Examples are Australia (Australian Prudential Regulation Authority, 2005), Hong Kong (Hong Kong Monetary Authority, 2006), the UK (Financial Services Authority, 2003, Bank of England, 2013), and the US (Office of the Comptroller of the Currency et al., 2003, 2007).

The Prudential Regulation Authority (PRA) of the Bank of England (see Bank of England, 2013) expects banks “[...] to ensure that no discount rate used to estimate LGD is less than 9%.” This 9% floor for the LGD discount rate was substantiated by a review that “[...] found that there was no widely-accepted industry approach to determining appropriate discount rates, insufficient evidence of the appropriateness of rates; and a tendency to reduce discount rates over time [...]”. The PRA accepts lower discount rates if conservative cash flows (e.g., certainty equivalent cash flows) are applied. Furthermore, the PRA is willing to consider alternative industry approaches subject to review.

Accounting standards differ from this interpretation as they prescribe the use of the effective rate. The effective rate is the rate that exactly discounts expected future cash payments or receipts through the expected life of the financial instrument to the initial investment. This is generally the contract rate for fixed rate loans and the current interest rate for floating rate loans. US banks have been subject to Statement of Financial Accounting Standards No.114 (Accounting by Creditors for Impairment of a Loan, FAS-114) which prescribes the effective interest rate as discount rate. For international accounting standards, IAS 39 prescribes the effective interest rate of the financial assets. In the future, larger banks will be subject to International Financial Reporting Standards (IFRS) established by the International Accounting Standards Board (IASB). In essence, IFRS 9 continues to prescribe the discount rate to be the effective interest rate. Handorf (2009) provides an overview of current accounting approaches.

In summary, prudential regulators are open to various approaches for LGD discount rates on economic substance provided they include the time value of money and a premium for non-diversifiable (systematic) risk. Contrary to this, accounting standards prescribe the use of the effective rate (i.e., the contract rate in most instances).

4.2 Guiding principles for LGD discount rates

We focus in this paper on discounted cash flow (DCF) methods as this matches the data environment in the banking industry. For example, GCD collects observed cash flows during work out on mostly non-traded instruments. In this technique, expected cash flows are discounted by risk adjusted discount rates. Observed LGDs are computed by discounting observed cash flows by the risk adjusted discount rate. The WG proposes the following guiding principles:

- Discount rates should be based on the opportunity costs of financial instruments with comparable price-relevant factors to avoid arbitrage opportunities for non-regulated market participants and hence market inefficiencies.
- Key price factors per regulatory guidance includes the risk free rate and premium for non-diversifiable risk. We interpret non-diversifiable risk as systematic risk that cannot be diversified in financial markets.
- Discount rates should be based on information that is available at the time of default. This includes parameter estimates based on historic information.
- Discount rates should exclude premiums for realized risk. Discount rates should reflect resolution risk only and thus (i) not include a premium for default risk and (ii) not exclude the premium for resolution risk.

This paper does not consider but encourages a more detailed analysis of the following aspects:

- Recoveries may be decomposed into assets (determined ex-ante) of different degrees of systematic risk. These assets may be appraised using discount rates that reflect the difference in systematic risk.
- Banks may apply conservative estimates for recovery realizations and a certainty equivalent method with the risk free rate as discount rate, as appropriate.
- The Basel Committee on Banking Supervision (2005a) mandates that “[...] appropriate risk premiums for the estimation of LGDs consistent with economic downturn conditions [...]” are applied. The Working Group notes that downturn LGD may not be clearly defined in a regulatory context.⁶ Therefore, banks should carefully consider any interaction between the (i) discount rate method, (ii) the downturn method and the data on which (i) and (ii) are based.
- The impact of taxation on cash flows and discount rates.
- [Term structure of interest rates.](#)

⁶ Downturn LGD may be inferred from analogy to the Downturn PD as computed by the Vasicek formula as part of the calculations of risk-weighted assets in internal ratings-based models under Basel III.

4.3 Review of discount rate approaches

The literature has proposed a number of competing approaches, which may be separated into loan contract specific, comparable and equilibrium approaches.

4.3.1 Loan contract rate

Asarnow & Edwards (1995) propose the loan contract rate of the defaulted loan as discount rate.

Default and resolution risk are priced in the contract rate. The annual **return given a non-default** is the contract rate k

$$r_{D=0} = \frac{(1+k) - 1}{k}, \quad (10)$$

With reference to Figure 1, we focus on default risk and set the outcome after default to ELGD and assume a loan with a maturity of one year and default at maturity. Assuming that interest (based on the contract rate) has accrued to a full period, the annual **return given a default** is

$$r_{D=1} = \frac{(1+k)(1-ELGD) - 1}{k(1-ELGD) - ELGD}. \quad (11)$$

Therefore, the **expected return** with regard to default risk is:

$$E(R_L) = (1-p)r_{D=0} + pr_{D=1} \quad (12)$$

$$= (1-p)k + p(k(1-ELGD) - ELGD)'$$

with the probability of default p and the random return on the loan (index L) R_L . Note that the expected return for resolution risk is embedded in ELGD in the above equations.

In contrast to the expected return with regard to resolution risk, the **contract rate** at origination includes the expected loss, and the LGD-adjusted likelihood of non-default (see also Chalupka & Kopecsni, 2008). This becomes apparent after solving Equation (12) for k :

$$k = \left(E(R_L) + \frac{pELGD}{\text{expected Loss}} \right) / (1 - pELGD). \quad (13)$$

The contract rate has two shortcomings: (i) it relates to the expected return with regard to default risk and resolution risk, which is set at origination,⁷ and (ii) it exceeds the expected return for the loan as it includes the expected loss. As a result, the contract rate contradicts the guiding principles presented in Section 4.2. Alternatively, the expected return implied in the contract rate from Equation (12) may be used as this is more closely aligned with the guiding principles.

⁷ The systematic risk may change post default with the realization of default risk. The Workgroup did not discuss this in more detail but encourages further research.

However, estimated expected returns imply a circular reference to estimated ELGD assumption, which require the knowledge of the discount rate. To avoid the circular reference, one may apply ENLGD for the realized return (p.a.) given a default:

$$r_{D=1} = ((1 + k)(1 - ENLGD))^{1/((T_D-0)+(T_R-T_D))} - 1, \quad (14)$$

The formula assumes that all cash flows are received at time to resolution and computes the internal rate from the resolution time to the loan origination.

A length of one period may be assumed for the time of loan origination to default ($T_D - 0$) for simplicity. The resulting expected return with regard to default risk and resolution risk is:

$$\begin{aligned} E(R_L) &= (1 - p)r_{D=0} + pr_{D=1} \\ &= (1 - p)k + p \left(((1 + k)(1 - ENLGD))^{1/(1+T_R-T_D)} - 1 \right), \end{aligned} \quad (15)$$

R_L is the return on the loan, which is based on the contract rate and hence only indirectly on financial markets. Furthermore, Office of the Comptroller of the Currency et al. (2003)⁸ propose the contractual rate of the highest risk grade. This proposal is very similar to using rates which may apply after covenant violations, or alternatively, the contract rate at default. Note that contract rates for high idiosyncratic risk exposures and prior or at default are likely to include the pre-default expected loss and a similar correction may apply.

Discount rate example: the estimated probability of default is 1%, the contract rate is 5%, ENLGD is 60%, and time to resolution is 2 years. Therefore, the estimated expected return (and hence discount rate) with regard to default risk is 4.7% following Equation (15)

$$E(R_L) = (1 - 0.01)0.05 + 0.01 \left(((1 + 0.05)(1 - 0.6))^{1/(1+2)} - 1 \right) = 0.0470.$$

4.3.2 Bank weighted average cost of capital (WACC)

Witzany (2009)⁹ and Jensen (2015) have proposed the application of the weighted cost of capital as a discount rate. Weights are generally based on the relative proportion of equity and debt funding of a

⁸ The Office of the Comptroller of the Currency et al. (2003) stipulate that “A bank must establish a discount rate that reflects the time value of money and the opportunity cost of funds to apply to recoveries and costs. The discount rate must be no less than the contract interest rate on new originations of a type similar to the transaction in question, for the lowest-quality grade, in which a bank originates such transactions. Where possible, the rate should reflect the fixed rate on newly originated exposures with term corresponding to the average resolution period of defaulting assets.” Interestingly, this approach is no longer found in the updated guidance Office of the Comptroller of the Currency et al. (2007).

⁹ Witzany (2009) presents initially a CAPM Model, which results in a WACC model for the discount rate and computes (iteratively) the spread as the capital risk charge times the regulatory capital for market risk.

bank in market value terms.

It is often argued that the bank funding costs do not reflect the risk profile of individual credit exposures with regard to resolution risk given loan default (compare e.g., Chalupka & Kopecsni, 2008). Furthermore, market funding costs for distressed/defaulted assets would be the preferred approach but very difficult to determine. Therefore, the WG has explored the computation of loan-specific capital and debt ratios based on the capital requirements of defaulted loans relative to the post-default expected loan value. Under the assumption that regulatory capital is a reasonable measure for systematic risk and that post-default capital and debt ratios are consistent with the ones of other (e.g., pre default) loan instruments, bank funding costs may provide a basis for a reasonable discount rate.¹⁰ Structurally, this approach combines bank-level funding costs and loan-level funding ratios.

With the default event, default risk is realized and the post-default loan value is assumed to equal the expected recovery:

$$V_{T_D} = (1 - ELGD)EAD \quad (16)$$

The post-default capital ratio e_{T_D} is:

$$e_{T_D} = \frac{E_{T_D}}{V_{T_D}} = \frac{DLGD - ELGD}{1 - ELGD}, \quad (17)$$

With the post-default equity value assumed to be equal to the unexpected loss given default:

$$E_{T_D} = DLGD - ELGD \quad (18)$$

The debt ratio d_{T_D} follows as $1 - e_{T_D}$.

We use index C for the expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk. This expected return may be based on the combination of the expected return on equity weighted by the post-default capital ratio and the expected return on debt weighted by the post-default debt ratio:

$$\begin{aligned} E(R_C) &= e_{T_D} E(R_E) + d_{T_D} E(R_D) = \\ &= \frac{DLGD - ELGD}{1 - ELGD} E(R_E) + \left(1 - \frac{DLGD - ELGD}{1 - ELGD}\right) E(R_D) \end{aligned} \quad (19)$$

In practice, the expected return on equity and debt may be approximated by equity and debt funding costs. The formula implies a circularity issue as ELGD and DLGD are required which in turn are in practice estimated from OLGD, which requires the discount rate.

¹⁰ Prudential regulators have increased the focus on consistency between the various regulations (see Basel Committee on Banking Supervision, 2015).

Discount rate example: the estimated probability of default is 1%, estimated ELGD is 60%, DLGD is 63.2%, the post-default capital ratio is $(63.2\%-60\%)/(1-0.6)=8\%$, the debt funding costs are 4% and the return on equity is 7.8%. Therefore, the estimated expected return (and hence discount rate) with regard to resolution risk is 4.30%:

$$E(R_c) = 0.08 * 0.078 + 0.92 * 0.04 = 0.0430$$

4.3.3 Bank return on equity

Eales & Bosworth (1998) discuss a range of discount rate approaches and decide to apply the return on equity as the discount rate. Their justification is that equity is debited/credited for differences between ELGD and OLGD and that the risk exposure of LGDs corresponds to the equity position. The approach may be interpreted as a special case (i.e., upper boundary) of the WACC approach where the capital ratio is 100%:

$$E(R_c) = E(R_E) \quad (20)$$

We use index C for the expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk. For example, the cost of equity may be computed using the capital asset pricing model:

$$E(R_E) = rf + \beta E(R_M - rf), \quad (21)$$

with the expected market excess return $E(R_M - rf)$ and $\beta = \frac{Cov(R_E, R_M)}{Var(R_M)}$. Averages for realized market excess returns may be taken as proxies for expected values and are reported for broadly based equity market indices in various geographies.¹¹ Other approaches such as using average share returns (after adjusting for dividend payments), bank ROE target numbers, or accounting ROE ratio may be considered.

Discount rate example: the bank equity beta to the market return is 0.8, the risk-free rate is 3% and the estimated expected market excess return is 6%. Therefore, the estimated expected return on equity (and hence discount rate) is 7.8%:

$$E(R_c) = 0.03 + 0.8 * 0.06 = 0.078.$$

¹¹ Depending on historic time period and geography, market risk premiums are between 2% and 8% p.a., see Dimson et al. (2011).

4.3.4 Market return of marketable credit instrument

Brady et al. (2006) and Jacobs Jr (2012) compute the realized returns and average realized returns for defaulted bonds. In a similar fashion returns on distressed debt (see Altman & Kuehne, 2012, who look more broadly at returns of distressed bonds) may be applied. The realized return (p.a.) given a resolution outcome b of a marketable financial instrument is:

$$r_{b,c} = \left(\frac{B(T_R)}{B(T_D)} \right)^{1/(T_R - T_D)} - 1, \quad (22)$$

with the instrument price B . We use index c for the return for an instrument that is comparable to a defaulted exposure subject to resolution risk. Note that it is common to use the bond price 30-45 days past default as a proxy for the bond price at default.

The expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk may be based on the average as an estimate for the expected return:

$$\begin{aligned} E(R_c) &= \frac{1}{B} \sum_{b=1}^B r_{b,c} \\ &= \frac{1}{B} \sum_{b=1}^B \left(\left(\frac{B_b(T_R)}{B_b(T_D)} \right)^{1/(T_{b,R} - T_{b,D})} - 1 \right) \end{aligned} \quad (23)$$

For simplicity, we have assumed that the prices of all B defaulted bonds are observed and that the market expectations are based on the mean of realized returns.

Discount rate example: the bond price at resolution for two defaulted bonds is 55 and 45, the time to resolution is one and two years, the bond price at default is 40 for both bonds. The realized annual return for the first bond is $(0.55/0.4)^{(1/1)} - 1 = 0.375$ and for the second bond $(0.45/0.4)^{(1/2)} - 1 = 0.0607$. Therefore, the estimated expected return (and hence discount rate) with regard to resolution risk is 21.78%:

$$E(R_c) = 1/2(0.375 + 0.0607) = 0.2178$$

4.3.5 Market equilibrium return

Maclachlan (2004) suggests basing discount rates on market equilibrium models such as the capital asset pricing model (CAPM). In these models, the expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk may be based on the expected return of an asset, which in turn is based on the risk free rate, the beta and the market risk premium (i.e., the return of the market in excess of the risk free rate):

$$E(R_E) = rf + \beta MP, \quad (24)$$

with the risk free rate rf , the beta $\beta = \frac{Cov(R_C, R_M)}{Var(R_M)} = \frac{Corr(R_C, R_M)Std(C)}{Std(R_M)}$, and the market risk premium $E(R_M - rf)$. We use index E as the expected return is based on an equilibrium model. The CAPM requires a number of assumptions such as frictionless financial markets that are controversial during economic downturns when default events are most likely to occur. Examples include the ability of market participants to borrow and lend, the absence of transaction costs and availability of all information to all market participants.

Discount rate example: the defaulted debt beta with regard to the market return is 0.5, the risk-free rate is 3% and the estimated expected market excess return is 6%. Therefore, the estimated expected return (and hence discount rate) with regard to resolution risk is 6%:

$$E(R_C) = 0.03 + 0.5 * 0.06 = 0.06.$$

Maclachlan (2004) estimates the defaulted debt beta by the:

- Correlation between the return on a defaulted bond index and defaulted bonds and the market return. The analysis results in defaulted debt betas of 36% based on the NYU Bond Index and 37.1% based on 90 defaulted US bonds. The market index was the S&P 500 index in both analyses.
- Asset correlations of 17% from Frye (2000),¹² resulting in defaulted debt betas of 30% based on an asset correlation. The beta coefficient may be computed as follows

$$\beta_{i,j} = \frac{\sigma_{i,j}\sqrt{AC_j}}{\sigma_{M,j}} \quad (25)$$

with borrower i and risk segment j .

Alternative approaches to determine $\beta_{i,j}$ from Equation (25) include distressed bond and equity prices of defaulted borrowers. However, such approaches are limited by the number of observation and the separation of systematic and idiosyncratic risk. The WG believes that further research should be undertaken. The following alternative approaches for the risk premium have been suggested by the work group:

- Regress recovery rates on equity market excess returns or Fama-French factors;
- Bank loan recoveries are correlated with GDP which in turn is highly correlated with consumer

¹² Note that Basel II/III provides asset correlations between 12% and 24% for corporate credit exposures in the Internal Ratings-based Approach.

consumption and the Consumption CAPM may provide for a price link.

Roesch & Scheule (2012) model empirical asset correlations AC_j by conditioning on historic average LGDs (e.g., through the cycle models) and time-varying information (e.g., point-in-time models).¹³ Assuming a normal distribution, the natural logarithms of observed recovery rates at a certain point in time ($ORR_{ijt}=1-OLGD_{ijt}$) may be described as follows:

$$\ln(ORR_{ijt}) = \alpha_j + \beta_j x_{jt} + \gamma_j \varepsilon_{jt} + \delta_j \varepsilon_{ijt}, \quad (26)$$

with borrower i , risk segment j and default time t . The parameters may be estimated risk segment-specific. α_j is an intercept, β_j the sensitivity to observable (in particular macro-economic) information, γ_j the sensitivity to a standard normally distributed systematic random effect and δ_j the sensitivity to an idiosyncratic standard normally distributed error term. γ_j and δ_j may also be interpreted as standard deviations. x_{jt} are time-varying systematic variables (here the lagged average log recovery and GDP growth). Other variables including loan-specific variables are possible. The model may be extended to control for selection issues by the consideration of default events and cure events (see Roesch & Scheule, 2012).

The asset correlation (AC) follows from Equation (26):

$$AC_j = \frac{\gamma_j^2}{\gamma_j^2 + \delta_j^2} \quad (27)$$

4.4 Benchmarking of discount rate approaches

The WG has discussed these concepts in light of a number of criteria:

- Guiding principles of discount rates: degree to which the approach is supported by the principles for discount rates established by the WG. WACC and equilibrium returns meet these properties, while the other approaches violate aspects of these principles:
 - Contract rate: the concept includes a compensation for default risk (in essence the credit spread), which has been realized post default. Furthermore, contract rates relate generally to the origination time, with regard to systematic risk and the time value of money;
 - Return on equity: the concept assumes that the systematic risk is equal to an equity

¹³ This is comparable to default risk modelling, where asset correlations have been measured by linking time-varying defaults and default rates to historic average default rates (e.g., TTC models). In extensions, frailty effects conditional on observable time-varying information (e.g., PIT models) have been estimated (compare, e.g., Hamerle et al., 2006).

- investment in the bank which is unlikely to be reasonable given the heterogeneity of defaulted loans. The bank-specific return on equity is unlikely to include an appropriate premium for the systematic resolution risk of the defaulted loan;
- Market returns of defaulted bonds: the concept is based on a small number of bond defaults and resulting returns are unlikely to provide for robust comparable discount rates.
 - Simplicity: the approach is based on available measurable information, which does not require assumptions in the measurement process:
 - Contract rates: are generally observable but may be unclear for credit lines, derivatives and guarantees;
 - WACC and return on equity: require a moderate level of assumptions;
 - Market returns of defaulted bonds: comparability of defaulted bonds and defaulted loan is unclear (in particular for SME loans);
 - Equilibrium returns: require strong assumptions for the estimation of measures for systematic risk and computation of risk premiums.
 - Application to GCD data:
 - Contract rates: are only observable in a small fraction of observations.
 - WACC and return on equity: GCD has no data on bank funding costs but may be used to compute the loan-level capital and debt ratio for WACC;
 - Market returns on defaulted bonds: GCD has no data on defaulted bonds;
 - Equilibrium returns: GCD data may be used to estimate the exposure to systematic risk.
 - Negative LGDs: the realized LGD is computed based on the book value of EAD (i.e., expected outstanding principal at default) and the market value of resulting post-default realized LGD if the current interest rates and current spreads for systematic risk are applied. Low interest rate regimes (e.g., as a consequence of monetary easing) in conjunction with high net recovery cash flows (e.g., in the instance of a cure) may result in present values in excess of EAD and hence negative LGDs. For example, a default followed by a loan service according to schedule and discount rates (determined at default) below the contract rate (determined at origination) would result in a recovery rate greater than one and hence a negative LGD. LGD values that are constrained within the interval $[0,1]$ may support general acceptance and the

input requirements of some Stage II regression models that require values between zero and one.

- Contract rates: LGDs are within the interval [0,1];
- All other concepts may result in negative LGDs. However, the likelihood for negative LGDs is small.

Table 2 summarizes the WG view on these approaches. The criteria are phrased in the positive and evaluation categories may be: ***/agree, **/neutral, and */disagree.

Table 2: Evaluation of approaches

Criteria	Contract rate	WACC	ROE	Bond return	Equilibrium return
Guiding principles of discount rates	*	***	*	*	***
Simplicity for:					
SMEs	***	**	**	*	*
Large corporates	***	**	**	**	*
Financial institutions	***	**	**	**	*
Credit lines, derivatives and guarantees	**	**	**	*	*
Application to GCD data	**	***	*	*	***
LGD values in [0,1]	***	**	**	**	**

Two approaches have a preference in the Working Group as they meet the guiding principles and can be applied in a data rich environment that is available to GCD members:

- WACC based on the assumption that post-default capital is a reasonable reflection of systematic risk;
- Market equilibrium return based on the assumption that the link between measure for systematic risk and sensitivity to market excess returns is reasonable.

5 Applying different discount rates to GCD workout data

5.1 Using GCD workout data for empirical discount rate studies

Global Credit Data is a not for profit initiative to help banks to measure their credit risk, owned by its 50 member banks across Europe, Africa, North America, Asia and Australia. GCD has collected the world's largest LGD/EAD database with a large number of defaulted facility observations totaling over €200 billion in all Basel asset classes.

This GCD study analyzes this database and is to our best knowledge the first to estimate discount rates from observed resolution information. The use of the GCD database for LGD discount rates has a number of merits:

- Discount rates are based on systematic risk. The large number of defaulted facilities in the GCD database allows for the measurement of systematic risk, i.e., the remaining non-diversifiable risk in a diversified portfolio.
- The data set is sufficiently large to form risk segments such as geographies and industries, which allows for an estimation of segment-specific systematic risk and hence the required risk premiums.
- Most discount rate approaches may be inferred from GCD data. We base the costs of equity on a bank beta of one and cost of debt on the risk free rate. As market returns on defaulted bonds between default time and resolution time are not recorded by GCD. The database mainly captures banking book data on non-traded bank instruments. Hence, the market return approach is not included in the empirical part of this study and we refer the reader to Jacobs Jr (2012).

5.1.1 Screening the data and applying data filters

The empirical analysis is based on data provided by GCD in June 2015. For the purpose of this analysis, member banks agreed to a selection of filter rules as shown in Table 3, which shows the number of defaulted loan facilities and borrowers after the application of various filter rules.

29,569 defaulted facilities in relation to 17,193 borrowers remain after the application of these filters. We analyze LGDs on the facility level. A robustness check reveals that there is only a minor difference between facility and borrower based mean LGD per default year as the majority of borrowers relate to a single facility. Note that it is possible that some banks consolidate multiple defaulted loans by the same borrower into a single facility.

Table 3: Number of defaulted facilities and borrowers after the application of filter rules

Filter rules	Facilities	Borrowers
Raw Dataset	123,577	64,140
Facilities with FC, LC and SME as borrower	103,089	52,240
Resolved facilities	88,966	45,916
Unsecured facilities	34,498	20,395
Defaulted in 2000-2013	32,521	19,294
Various robustness checks ¹⁴	31,806	18,924
Borrower EAD > 10K Euro	29,569	17,193

5.1.2 Risk segmentation

In terms of risk segmentation, the following risk segments are being formed in terms of geographical and cultural proximity:

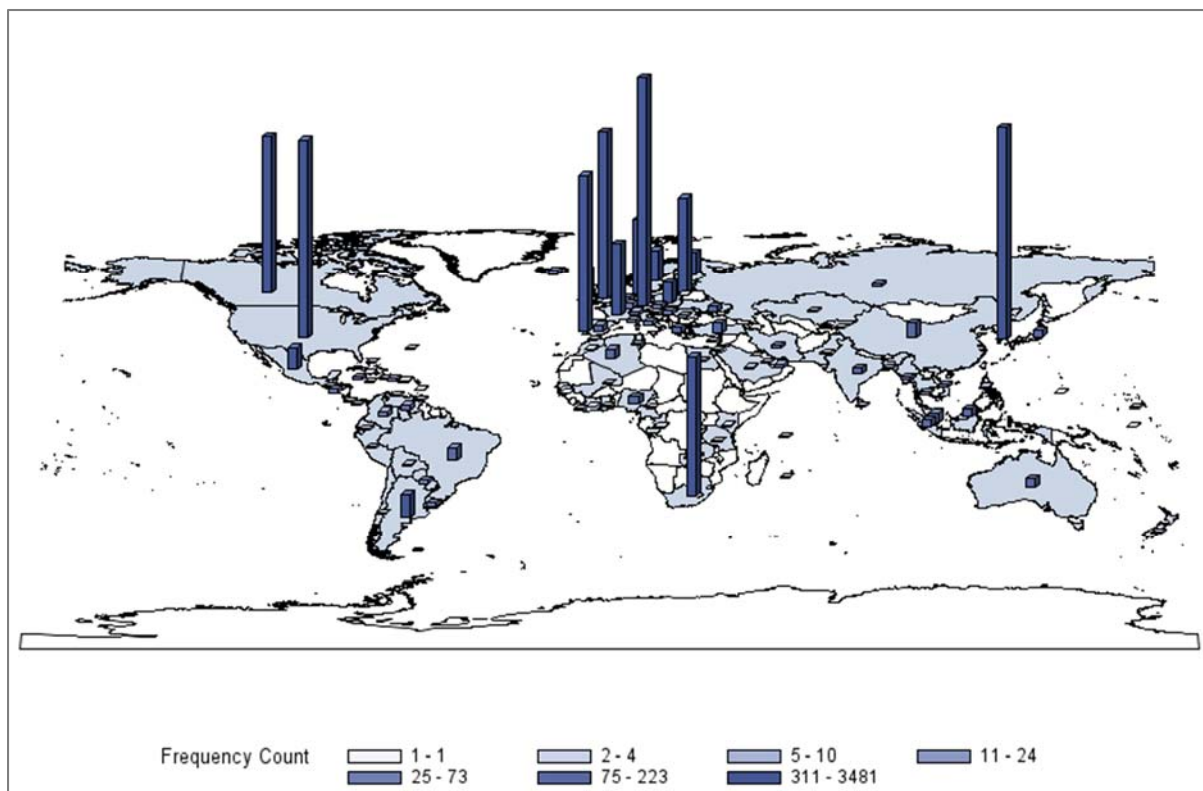
- Great Britain and Ireland;
- Central Europe¹⁵: Austria, Belgium, France, Germany, Luxemburg, Netherlands and Switzerland;
- Hispania: Portugal and Spain;
- North America: Canada and United States
- Scandinavia: Denmark, Finland, Norway and Sweden;
- South Africa;
- Others: all other countries.

¹⁴ These checks include: (i) a test to see whether the amount of write off and cash flows is reasonable, (ii) loan reported 'Resolved' showing all transactions in excess of Exposure by less than or 10 %, (iii) variable 'Entity Asset Class' must be given, (iv) if the Facility Asset Class = SME or Large Corporate, then the Entity Asset Class must equal Corporate, (v) if the Facility Asset Class equals 'Banks & Fin Co', then the Entity Asset Class must equal Banks or 'Non-bank Financial Company', (vi) Entity Asset Class must equal Banks or 'Non-bank Financial Company' when Primary Industry Code equals 'Finance and Insurance', (vii) Facility Asset Class must be given, (viii) Facility Asset Class must equal 'Banks & Fin Co' when Primary Industry Code equals 'Finance and Insurance', (ix) Loan Status must be given in the History table at least once for each Facility ID.

¹⁵ The working group is aware that Central Europe may be defined differently. We label Central Europe a set of geographies which the WG believes features risk characteristics that are close to each other.

Figure 2 shows the number of defaulted facilities after data filtering by country on a world map.

Figure 2: Total number of observations, by country

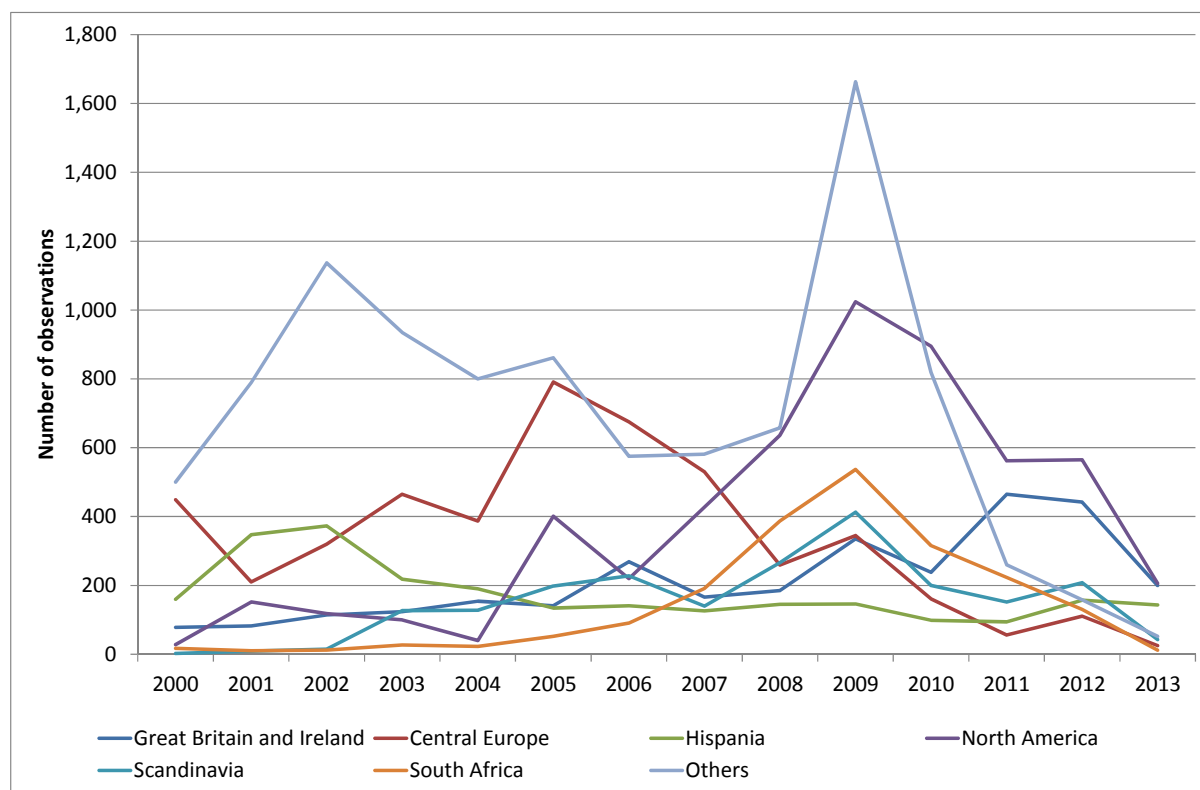


We have chosen the following risk segments in terms of industry segmentation:

- Commerce: Wholesale and Retail Trade;
- Construction;
- Finance: Real Estate and Rental and Leasing, Finance and Insurance;
- Manufacturing;
- Services: Transportation and Storage, Professional, Scientific and Technical Services, Education, Extra-Territorial Services and Organizations, Health and Social Services, Hotels and Restaurants, Other Community, Social and Personal Services, Private Sector Services (Household);
- Others: Agriculture, Communications, Hunting and Forestry, Fishing and Fishing Products, Mining, Public Administration and Defence, Utilities.

We have chosen the categories so that we observe on average at least 100 loss events per default year. Figure 3 shows the number of observations per risk segment:

Figure 3: Total number of observations by geography risk segment



The reason for this segmentation strategy is that we measure the exposure to systematic risk based on the time variation of loss rates. This requires the assumption that loss rates are not exposed to idiosyncratic risk for which we require on average a minimum of 100 loss observations per year.

The consequence of defining more granular risk segments¹⁶ would be that the observed loss rate variation is based on a combination of systematic and idiosyncratic risk and the exposure to systematic risk is overestimated.

¹⁶ One example would be to break out and combine the three industries "Agriculture, Hunting and Forestry", "Fishing and Fishing Products" in one category. This would imply a total of 904 loss observations (from 2000 to 2013) and 20-30 loss events per year in economic upturns and 100-2000 loss events in economic downturns.

5.1.3 Empirical strategy

The GCD data collects a number of information tables. We applied the following databases to set filter rules:¹⁷

- **Entity:** general information on the borrower or guarantor;
- **Loan:** general information on the facility;
- **History:** dates for five events: origination, one year prior to default, default, post default and resolution.

We have applied the following data set to obtain the timing of origination, default and resolution:

- **History:** dates for five events (see above).

We have applied the following data set to obtain the risk free GCD discount rate:

- **Loan:** general information on the facility.

We have applied the following data set to obtain the gross resolution cash flows:

- **Transaction:** transaction type, date and transaction (amount) cash flows.

In the data analysis, we follow a number of consecutive steps:

1. Application of filter rules;
2. Risk segmentation: all facilities are grouped into geographic and industry classes;
3. Computation of nominal LGD based on the LGD 2 definition and a zero discount rate (LGD based on risk free rate is provided in the GCD database),¹⁸
4. Correction of risk free and nominal LGD based on risk free rate and zero discount rate for time to resolution bias;
5. Estimation of ENLGD as the long run average over ONLGD and ELGD as the long run average over OLGD;
6. Merging of macro-economic risk drivers: real GDP growth and lagged mean log recoveries (from default to resolution time);
7. Computation of systematic risk measures as dispersion of a frailty effect, which models the deviation of the log recoveries from the long run average based on observed log recoveries;
8. Computation of contract rate (Contract) and pre-default expected return (Contract2);

¹⁷ We did not apply the remaining GCD data sets, which are Financial: entity financial (sales, assets and debt), Guarantor: guarantor credit risk information, and Collateral: general information on collateral.

¹⁸ The LGD 2 definition includes the sum of present values of all cash flows, excluding principal advances and financial guarantees, except write-offs and Interest Accruals, which are not cash flows. Please refer to Global Credit Data (2015).

9. Computation of cost of equity (ROE) and the weighted average cost of capital (WACC);
10. Computation of equilibrium returns (Equilibrium);
11. Computation of observed LGDs based on the discount rates;
12. Correction of observed LGD for time to resolution bias;
13. Production of statistics for discount rate analysis (Section 5.2) and LGD analysis (Section 0).

5.2 Empirical discount rate analysis

We analyze the risk free rate (GCD), contract rate (Contract), adjusted contract rate (Contract 2), weighted average cost of capital (WACC), return on equity (ROE), and equilibrium return (Equilibrium).

5.2.1 Assumptions

We follow the computations of Section 4.3 and make the following assumptions:

- Risk free rate (GCD): based on the three month EURIBOR rate and the short-term interest rates of the respective country. Note that EURIBOR is an interbank lending rate for unsecured funds and includes an average bank credit spread.
- Contract rates (Contract): contract rates are observable for approximately 13% of all observations at facility origination. The majority (74%) of facilities that have pricing information are priced with a floating rate. We extract the spread in excess of the base rate for floating rate loans and compute the contract rate as the sum of the spread and the risk free rate at default. We replace missing values by median contract rates (for both the contract rate and the pre-default expected return) by geography and default year.
- Adjusted contract rate (Contract 2): we consider the contract rate and the estimated pre-default expected return. The discount rate is calculated following Equation (15). The mean discount rate is 3.72% and an alternative computation using Equation (12) has resulted in similar discount rates: 3.46% if ELGD is estimated based on the risk free rate and 3.84% if ENLGD is used to avoid the circular reference. The economic impact on LGDs is comparable.
- Weighted average cost of capital (WACC): we assume for the Downturn LGD (which is an input to the WACC concept) the maximum average LGD over default years and by geography. This may be similar to conditioning on historic adverse macroeconomic states. Banks may apply alternate downturn LGD models. The discount rate is calculated following Equation (19). We avoid circular references by estimating ELGD as the mean of the observed LGD for a given risk segment based on the risk free rate. Robustness checks using ENLGD resulted in comparable results.

- Return on equity (ROE): a bank beta measure of one is assumed, the discount rate is calculated following Equation (21).
- Equilibrium return (Equilibrium): the discount rate is calculated following Equation (24). The beta measure is based on Equation (24) and Equation (25). Table 4 shows the resulting parameter estimates for the point-in-time regression for log recoveries applying segmentation by geography and a segmentation by industry:

Table 4: Parameter estimates for a PIT regression model for log recoveries with frailty effects

Segment	Int.	GDP	Avg. log RR	gamma	delta	AC	beta
Gr Britain and Ireland	-0.4319	-0.0518	0.3909	0.3811	1.1413	0.1003	0.5631
Central Europe	-0.3820	-0.0005	0.2437	0.1553	1.1084	0.0193	0.2467
Hispania	-0.2360	0.0327	0.2828	0.2389	0.9493	0.0596	0.4339
North America	-0.3548	0.0182	0.2389	0.2224	0.6593	0.1022	0.5682
Scandinavia	-0.5347	-0.0068	-0.0594	0.1619	0.9092	0.0307	0.3116
South Africa	-0.1793	-0.0004	0.7120	0.2099	0.9876	0.0432	0.3696
Others	-0.5961	0.0079	0.0475	0.1647	1.1726	0.0193	0.2473
Commerce	-0.2194	-0.0014	0.6112	0.1622	1.0413	0.0237	0.2737
Construction	-0.2215	-0.0073	0.3605	0.1604	0.8721	0.0327	0.3216
Finance	-0.3312	-0.0017	0.3243	0.1721	0.9375	0.0326	0.3210
Manufacturing	-0.2552	0.0027	0.4623	0.0942	0.9966	0.0089	0.1673
Services	-0.2406	-0.0030	0.5011	0.1580	0.9981	0.0245	0.2780
Others	-0.2138	-0.0032	0.7037	0.1131	1.1904	0.0089	0.1681

Note: This table provides regression results in accordance with Equation (26).

In the empirical analysis we have applied the reference values from Maclachlan (2004) $\sigma_j = 0.32$ and $\sigma_M = 0.18$ in conjunction with the AC estimates from Table 4 to estimate betas and discount rates and compute the LGDs given these discount rates. In this model, we include two systematic variables real GDP growth and mean log recovery, which are lagged by one period. Both variables have been shown to be powerful systematic control variables for default risk (see e.g., Lee et al., 2016). We avoid circular references by computing ORR as 1-OLGD based on the risk free rate. Robustness checks using ONLGD resulted in comparable results.

All discount rates are based on the combination of risk free rate and a spread:

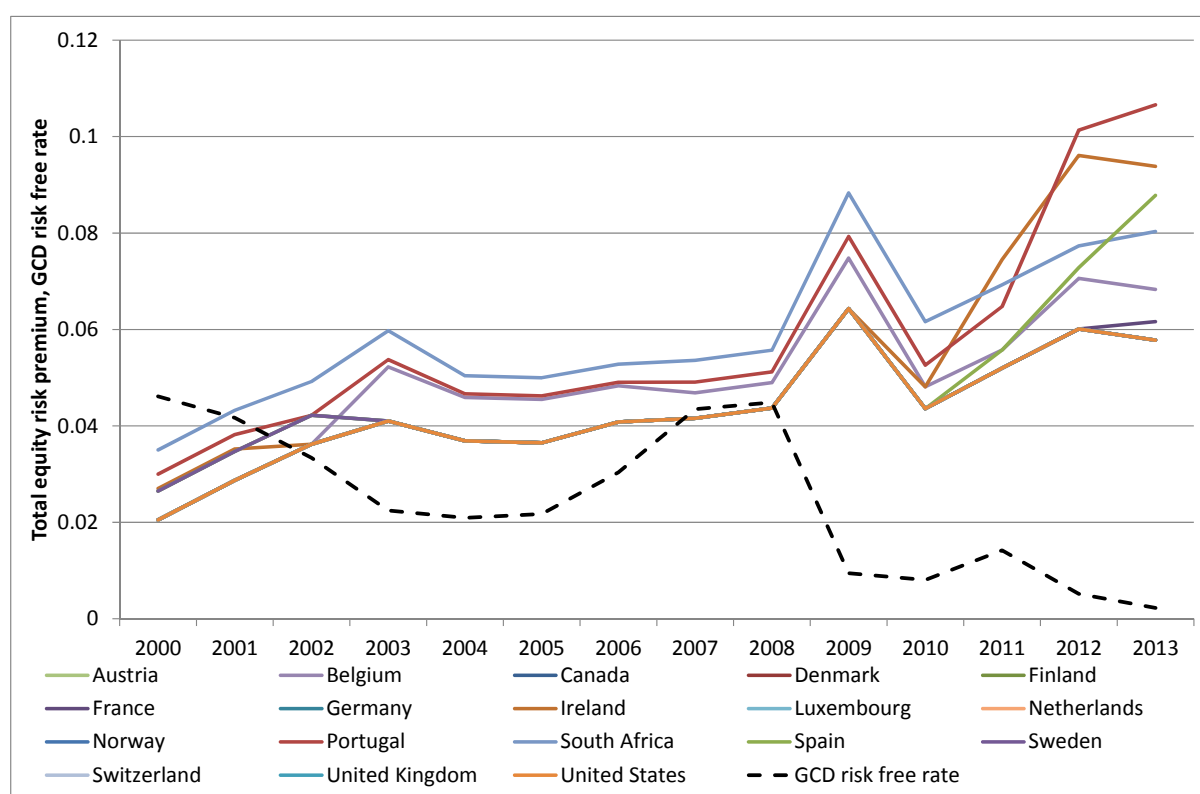
- GCD: risk free rate at default;
- Contract rate: risk free at default plus credit spread at origination;
- Contract 2: risk free rate at default plus credit spread at origination less expected loss;
- ROE: risk free rate at default plus equity risk premium at default times beta (assumed to be equal to unity);

- WACC: capital ratio times ROE at default plus debt ratio times risk free rate at default;
- Equilibrium: risk free rate at default plus equity risk premium at default times beta based on the systematic variation of mean LGDs over time (see next section).

The spread for ROE, WACC and Equilibrium approaches is based on the equity risk premium.

Whilst the CAPM theory does not provide for risk premiums outside the co-movement with the market return, the literature has pointed out that more volatile financial markets attract a greater risk premium, which we include into our analysis (see Damodaran, 2015). Other factors for risk premiums (e.g., premiums for size where smaller firms attract a higher risk premium) may exist but we do not include this aspect into our analysis as our knowledge of the borrowers underlying the defaulted facilities is limited.

Figure 4: Total equity risk premium by country and GCD risk free rate



The risk free rate is based on the GCD discount rate and the implied equity risk premiums are from Damodaran (2015), who publishes annual country risk premiums based on an average risk premium (RP) and the time-varying risk premium implied by future dividends (IRP) for the US (see <http://pages.stern.nyu.edu/~adamodar>). We computed an implied country risk premium on a country level by:

$$IRP_{kt} = RP_{kt} - RP_{US,t} + IRP_{US,t} \quad (28)$$

For example, for Canada in 2000 we observed the following values: $RP_{Canada,t} = 0.0611$, $RP_{US,t} = 0.0551$, and $IRP_{US,t} = 0.0205$. We compute the IRP for Canada as follows:

$$0.0265 = 0.0611 - 0.0551 + 0.0205$$

We define the index for a country as k (which is different from the broader risk segmentation j used before). The assumption behind the computation is that the US market (which is the largest in size) provides a base equity risk premium for national risk premiums. Figure 4 shows the total equity risk premium as the sum of a global market risk premium and a country risk premium (colored solid lines) and the risk-free GDC discount rate (black dashed line). Equity risk premiums and risk free rates offset and may mitigate the time variation of discount rates during this period.

5.2.2 Euro risk free rates

In this section, we use Euro as a pivot currency. Transaction amounts and amounts at default are converted and expressed in Euro. The LGDs are then calculated using those converted amounts. Consistently with the Euro denominated cash flows, the EURIBOR rate is used as a risk free rate for all facilities regardless of geography.

Figure 5: Median discount rates over time, observations with recorded contract rate

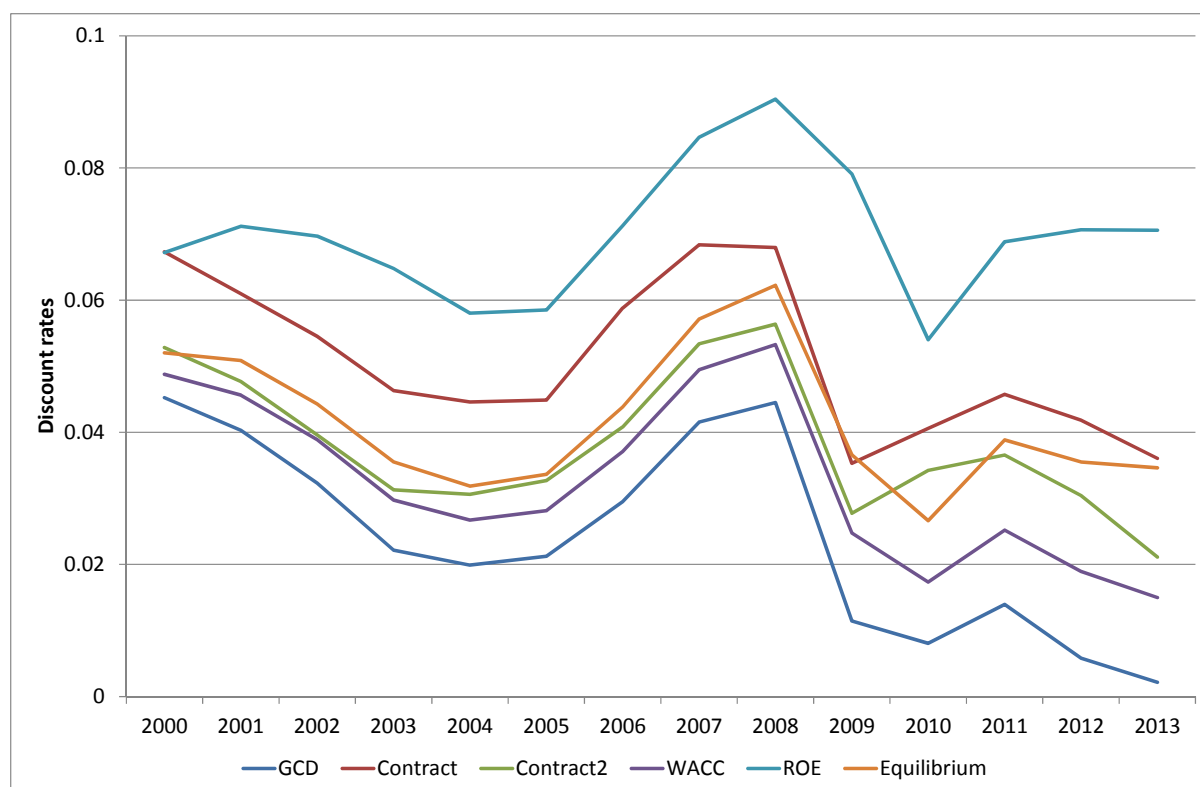


Figure 5 shows the resulting discount rates. The lowest discount rate is the risk free rate (GCD) and the highest discount rate is the ROE. Contract rates and market-implied discount rates are consistent, which is a reflection of the integration of lending and capital markets. Table 5 shows the moments for the main discount rates.

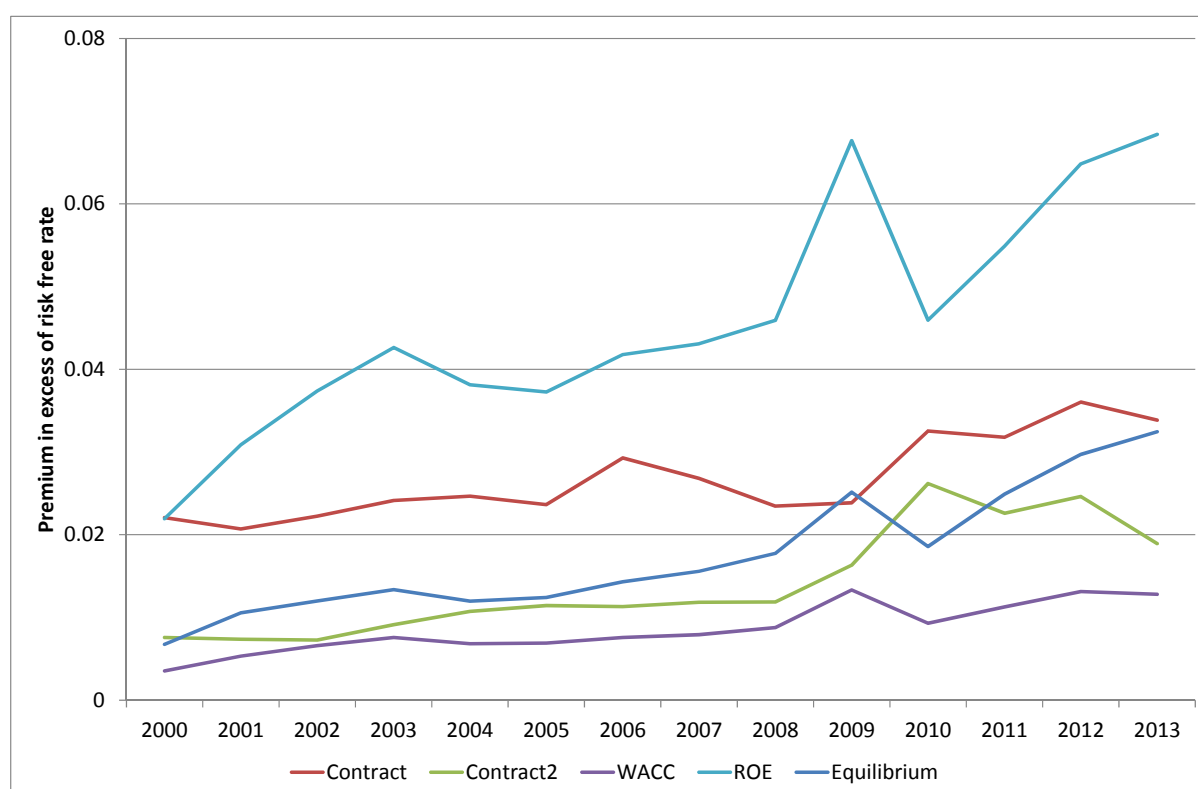
Table 5: Descriptive statistics discount rates

Measure	GCD	Contract	Contract2	WACC	ROE	Equilibrium
N	29,569	2,560	2,560	29,569	29,569	29,569
Min	0.0000	0.0000	-0.0275	0.0027	0.0205	0.0051
Max	0.0539	0.2762	0.1946	0.0647	0.1168	0.0782
Mean	0.0236 [#]	0.0491	0.0372	0.0326	0.0707	0.0413
StdDev	0.0147	0.0229	0.0214	0.0131	0.0136	0.0136

For the avoidance of doubt, in this table, 0.0236 is to be read as 2.36%.

Figure 6 shows the mean premiums in excess of EURIBOR for and the pre-default expected return for, WACC and ROE. The mean risk premiums increase during the reference period.

Figure 6: Median premiums in excess of EURIBOR over time, joint cross-section, observations with recorded contract rate



All risk-adjusted approaches are directly (equilibrium approach) or indirectly (other approaches) based on market prices for the time value of money (risk free rate), systematic risk (market risk premium) and exposure to systematic risk. This implies that loan exposures with a higher risk free rate, a higher

market price for systematic risk or a higher exposure to systematic risk result in higher discount rates. High levels of default risk for a borrower, an industry, or a country do not necessarily imply a high discount rate as a high level of idiosyncratic risk may not be causal for a high level of systematic risk and a high market price. As an example, Figure 4 shows that the sovereign rating downgrades in Southern Europe has a moderate impact on the market risk premium.

5.2.3 Currency-specific risk free rates

The previous analysis has used Euro as a pivot currency. The following figure shows the short-term interest rates which co-move for GCD countries. This includes South Africa (currency ZAR, top line) which has a higher level.

Figure 7: Mean discount rates by currency

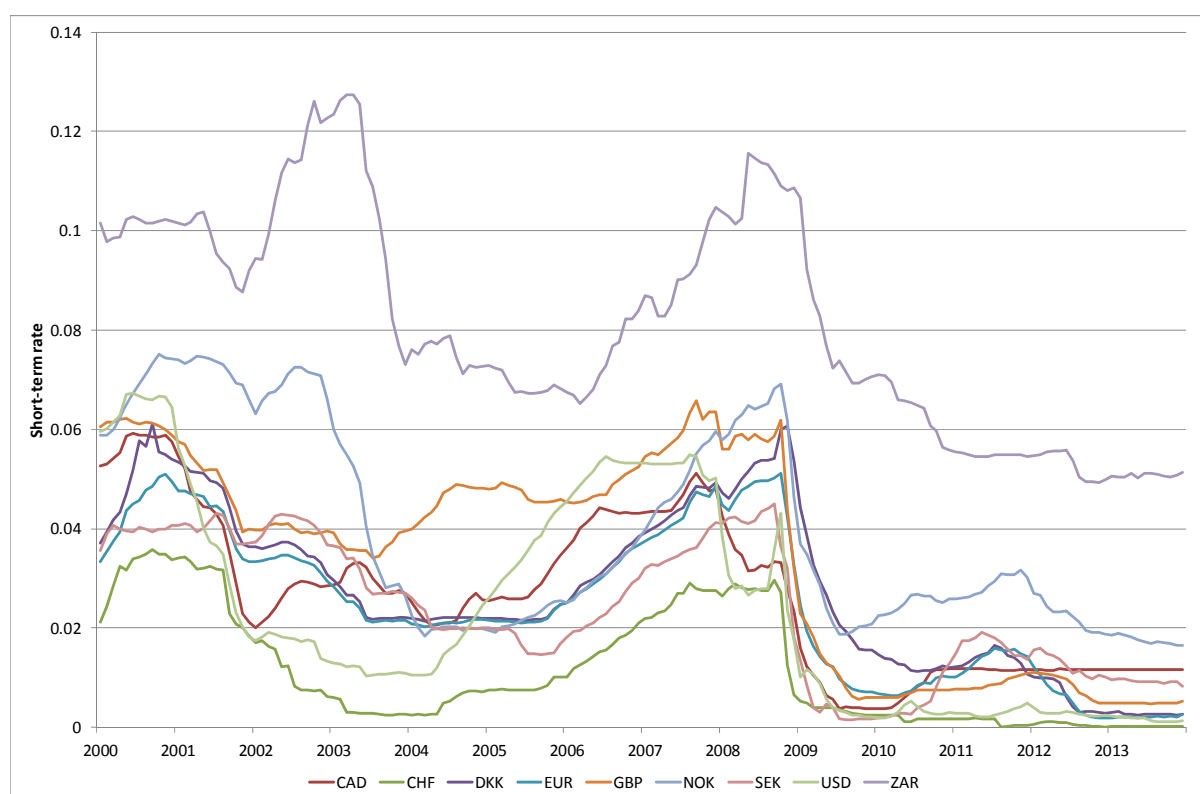
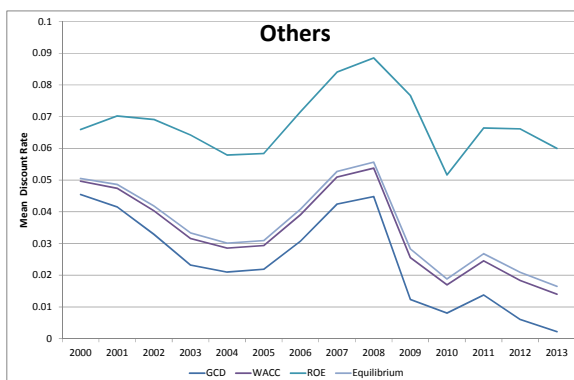
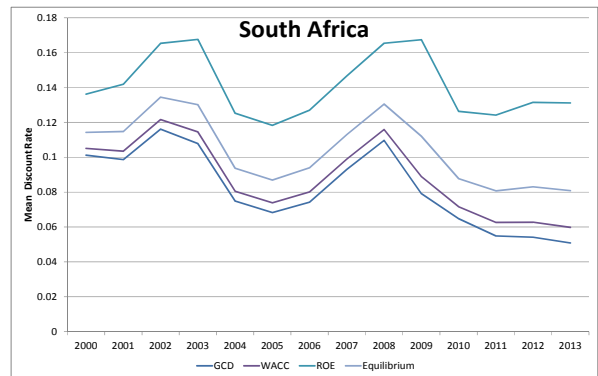
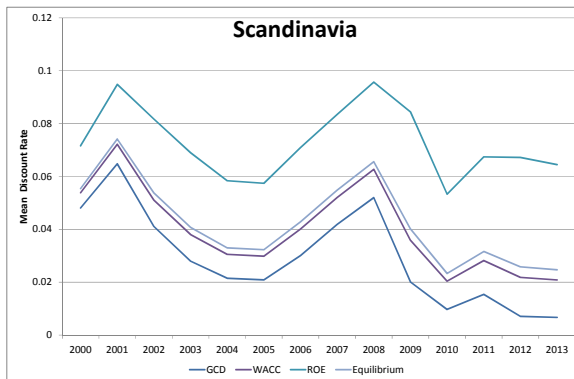
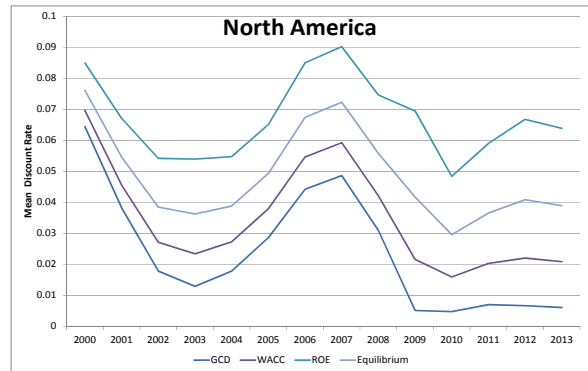
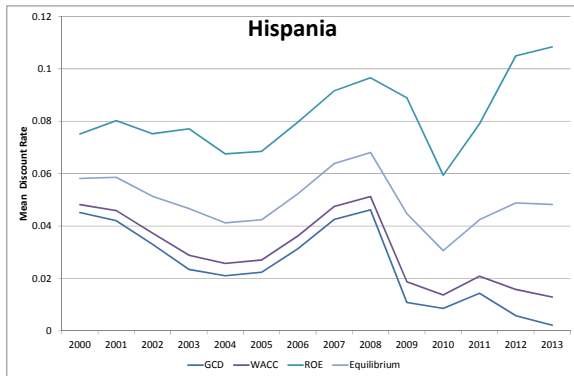
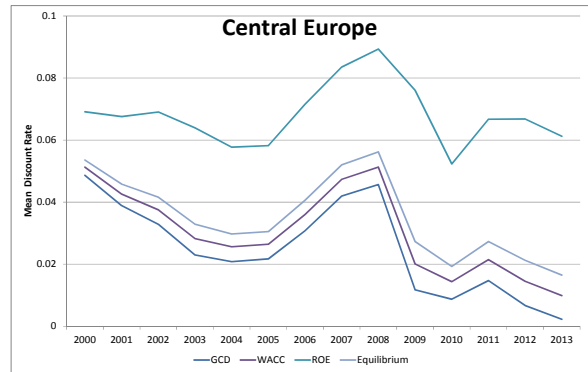
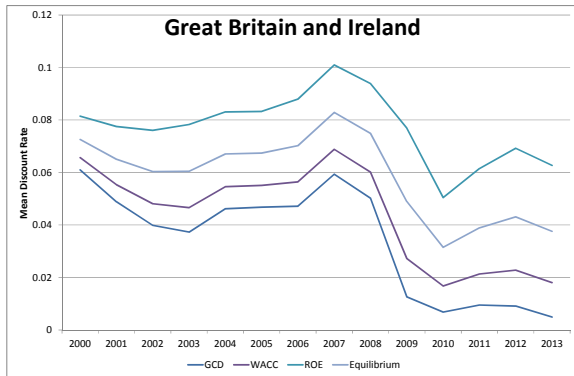


Figure 8 shows the mean discount rates by geography. We do not report the contract rate as the number of observations is insufficient.

Figure 8: Mean discount rates by geography



5.3 Empirical LGD analysis

5.3.1 Euro risk free rates

In this section we compute and compare the LGDs that result from the various discount rate concepts without further corrections for time to resolution bias. Resolution bias will be scrutinized in the next section.

Figure 9: Mean LGD (in %), without correction for resolution time bias

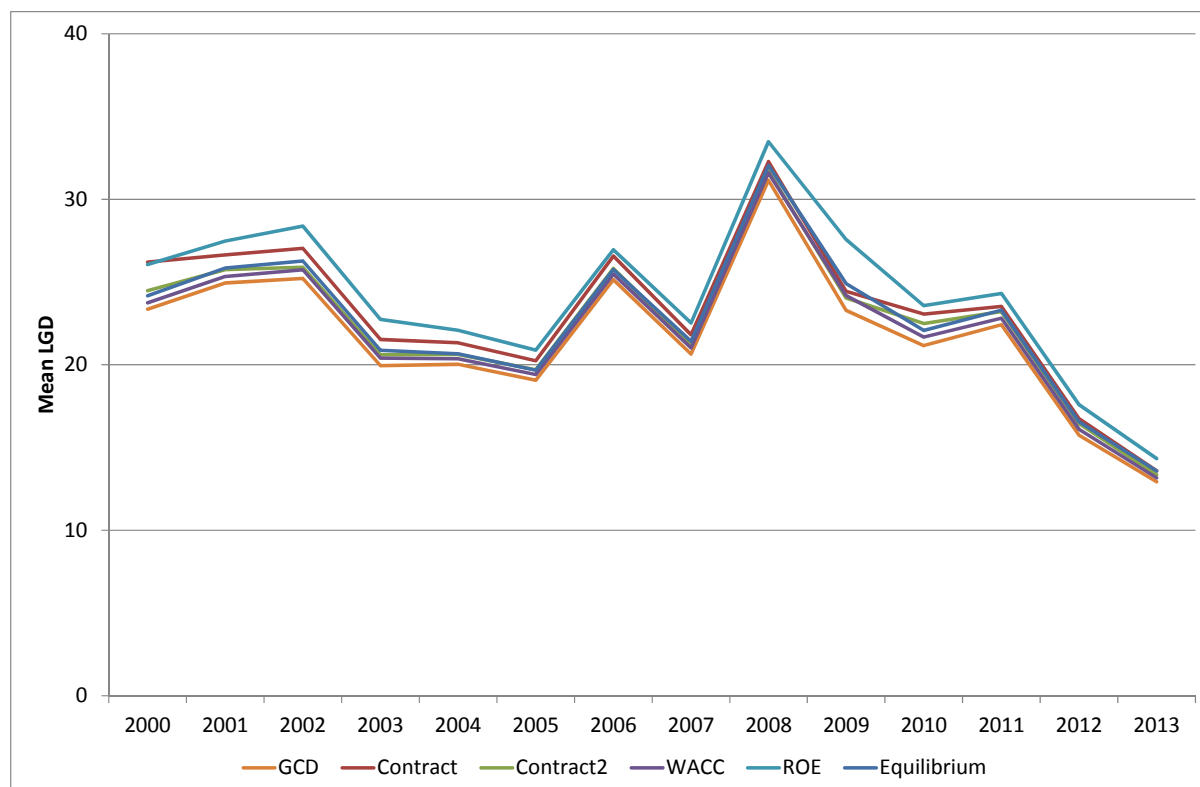


Figure 9 shows the mean LGD without correction for resolution time bias. Consistent with our prior expectations, a lower discount rate results in a lower LGD. The variation of LGDs given different discount rate concepts is limited and decreases with time to resolution in recent years. Table 6 shows the moments for the mean LGD, without correction for resolution time bias.

Table 6: Descriptive statistics LGDs (in %), without correction for resolution time bias

Measure	GCD	Contract	Contract2	WACC	ROE	Equilibrium
N	29,569	29,569	29,569	29,569	29,569	29,569
MIN	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MAX	150.0000	150.0000	150.0000	150.0000	150.0000	150.0000
MEAN	22.4122 [#]	23.8168	23.1600	22.8887	24.9517	23.3500
STD	35.9819	35.6135	35.7517	35.8369	35.3605	35.7172

[#] For the avoidance of doubt, in this table, 22.4122 is to be read as 22.4122%.

5.3.2 Correction for time to resolution bias

The data sample is subject to a resolution bias as unresolved LGDs with a higher time to resolution are not analyzed. Empirically, the time to resolution and LGD are positively correlated. To correct for this bias, we adjust the mean LGDs ($LGD_{t,adjusted}$) as follows:

$$LGD_{t,adjusted} = CR_t * LGD_{t,resolved} + (1 - CR_t) * LGD_{t,unresolved} \quad (29)$$

Table 7: Resolved LGD (in %), completion rate, estimated unresolved LGD and adjusted LGD.

Default Year	Obs.	Resolved LGD	CR	Unresolved LGD	Adjusted LGD
2000	1,234	23.3581	0.9724	70.6098	24.6613
2001	1,599	24.9387	0.9423	70.6098	27.5762
2002	2,089	25.2126	0.9368	70.6098	28.0830
2003	1,996	19.9498	0.9541	70.6098	22.2746
2004	1,722	20.0275	0.9498	70.6098	22.5664
2005	2,578	19.0705	0.9457	60.5835	21.3243
2006	2,199	25.1420	0.9524	47.5273	26.2085
2007	2,161	20.6456	0.9134	44.3458	22.6991
2008	2,536	31.1533	0.8493	42.3420	32.8395
2009	4,463	23.2844	0.8780	41.8823	25.5529
2010	2,728	21.1609	0.8289	40.1759	24.4139
2011	1,812	22.4168	0.7451	36.9915	26.1324
2012	1,771	15.7451	0.6856	35.1343	21.8404
2013	681	12.9344	0.6329	31.9048	19.8985

For the avoidance of doubt, in this table, 23.3581 is to be read as 23.3581%.

CR_t is the completion rate, which is the fraction of defaulted loans that have been resolved for a given default year. $LGD_{t,resolved}$ is the mean resolved and hence observed LGDs. We estimate the mean of unresolved (and hence unobserved) LGDs for a given default year by the mean over earlier resolved LGDs with a time to resolution greater than the observed LGDs for a given default year:

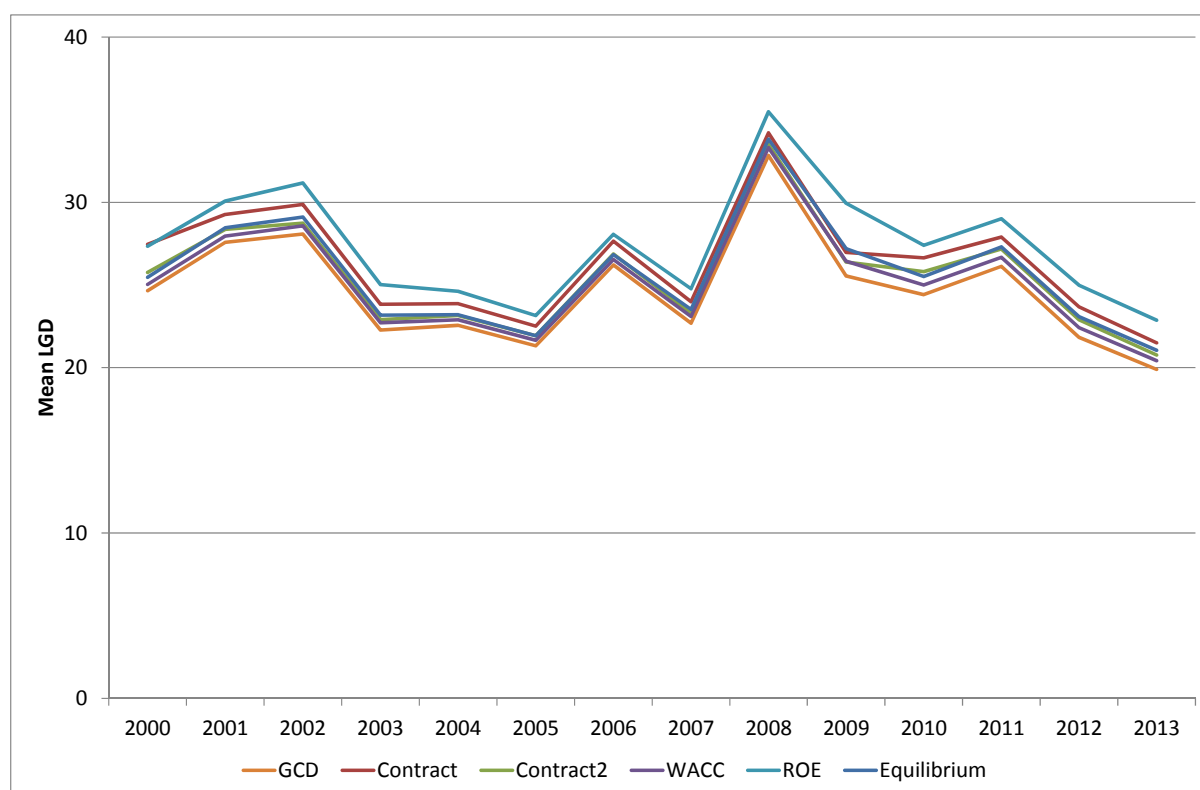
$$LGD_{t,unresolved} = \frac{1}{\sum_{i=1}^N I(TTR_{it,resolved} \geq 2015.5 - t)} \sum_{i=1}^N I(TTR_{it,resolved} \geq 2015.5 - t) LGD_{it,resolved} \quad (30)$$

Table 7 shows by default year the resolved LGD before correction, completion rates, estimated unresolved LGD and adjusted LGD after correction (LGDs are based on the risk free rate, GCD).

For example, given $LGD_{2013,resolved}=12.93$, $CR=63\%$, $t=2013$, end of observation period is 2015.5. The mean LGD of $TTR \geq 2.5$ is 31.90. The adjusted mean LGD is computed as follows: $LGD_{2013,adjusted} = 12.93 * 0.63 + 31.90 * (1 - 0.63) = 19.90$.

Figure 10 shows the mean LGD with correction for resolution time bias:

Figure 10: Mean LGD (in %), with correction for resolution time bias



The mean LGDs are higher than in Figure 9 over all default years as the decreasing completion rate (from 2000 to 2013) is offset by a decreasing implied LGD (from 2000 to 2013) which is higher than the observed LGD in all instances.

The choice of discount rates has a low to moderate impact, which is a function of the following two aspects:

- Average resolution time which is decreasing from 3.16 in 2000 to 0.51 in (2013)
- Average equity risk premium: the price for systematic risk is increasing from 2.19 in 2000 to 6.84% in 2013

These findings are in line with a paper by Gibilaro and Mattarocci (2011) that analyzes the impact of the risk free rate, contract rate and two equilibrium models (one based on the beta between average recoveries and GDP and average recoveries and a defaulted bond index). The average LGDs vary between 0.5051 (using a risk free discount rate) and 0.5327 (using the equilibrium model based on the defaulted bond index). Table 8 shows the descriptive statistics for the main discount rates for all years, the start year (2000) and the end year (2013).

Table 8: Descriptive statistics LGDs (in %) after adjustment for resolution bias

All years							
Measure	GCD	Contract	Contract2	WACC	ROE	Equil	Equil2
N	29,569	29,569	29,569	29,569	29,569	29,569	29,569
MIN	1.9475	1.9848	1.9582	1.9524	2.0025	1.9432	1.9648
MAX	147.8104	147.8477	147.8211	147.8153	147.8654	147.8061	147.8277
MEAN	25.0829 [#]	26.6813	25.9287	25.5887	27.9014	25.3255	26.1114
STD	31.7808	31.4584	31.5755	31.6553	31.2335	31.7091	31.5493
2000							
Measure	GCD	Contract	Contract2	WACC	ROE	Equil	Equil2
N	1,234	1,234	1,234	1,234	1,234	1,234	1,234
MIN	1.9475	1.9848	1.9582	1.9524	2.0025	1.9432	1.9648
MAX	147.8104	147.8477	147.8211	147.8153	147.8654	147.8061	147.8277
MEAN	24.6613	27.3862	25.7986	25.0397	27.3447	24.5213	25.4740
STD	30.5928	30.4993	30.4002	30.5786	30.6137	30.5729	30.5627
2013							
Measure	GCD	Contract	Contract2	WACC	ROE	Equil	Equil2
N	681	681	681	681	681	681	681
MIN	11.7122	12.8886	12.3366	12.0870	13.7965	11.8924	12.4638
MAX	77.3427	78.5057	77.9596	77.7118	79.4042	77.5225	78.0872
MEAN	19.8985	21.5108	20.8231	20.4184	22.8705	20.1990	21.0495
STD	19.9879	19.8625	19.9010	19.9433	19.7503	19.9484	19.8662

For the avoidance of doubt, in this table, 25.0829 is to be read as 25.0829%.

Figure 11: Mean LGD (in %), with correction for resolution time bias, by geography, EUR discount rate



Figure 12: Mean LGD (in %), with correction for resolution time bias, by industry, EUR discount rate

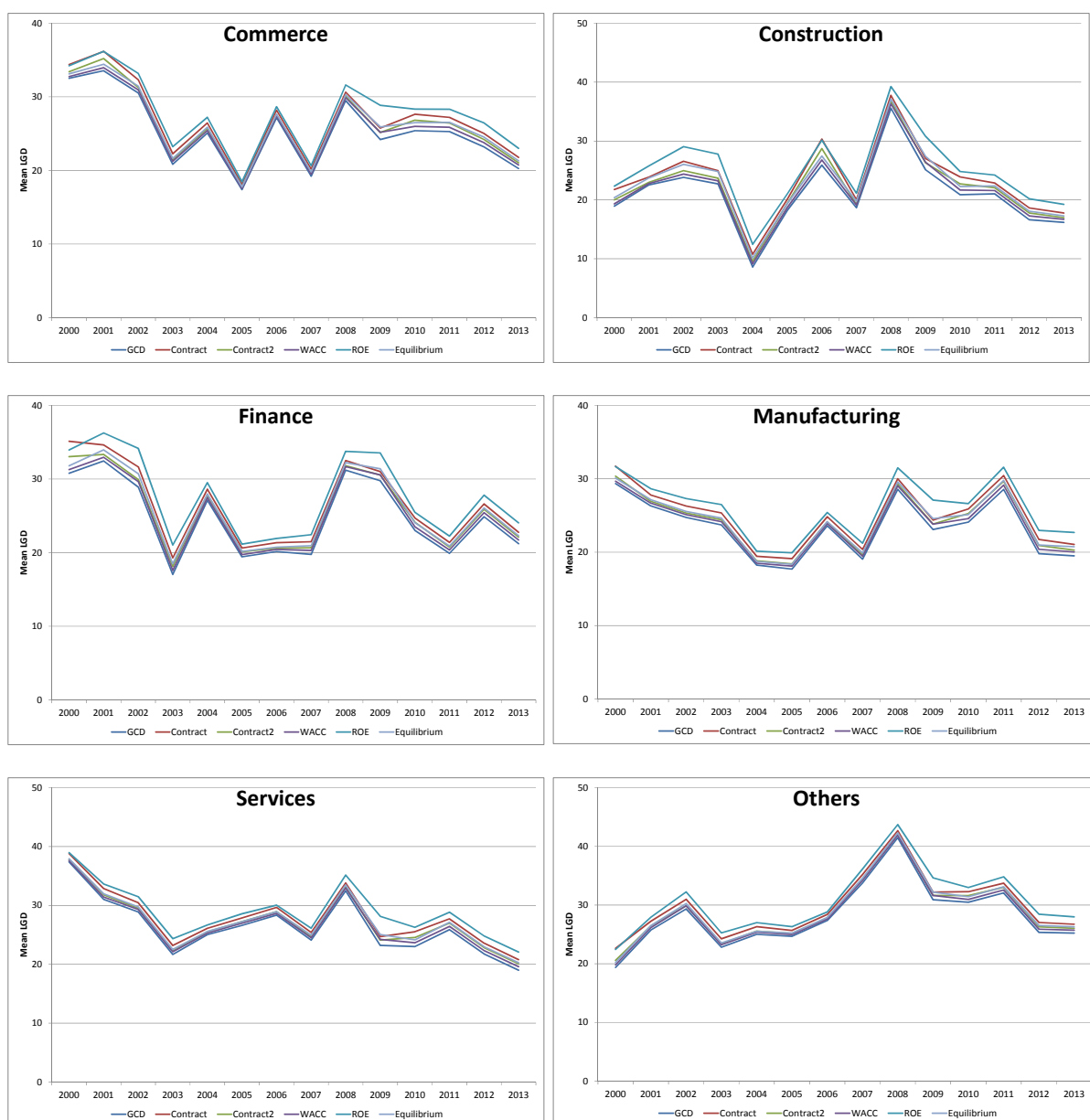
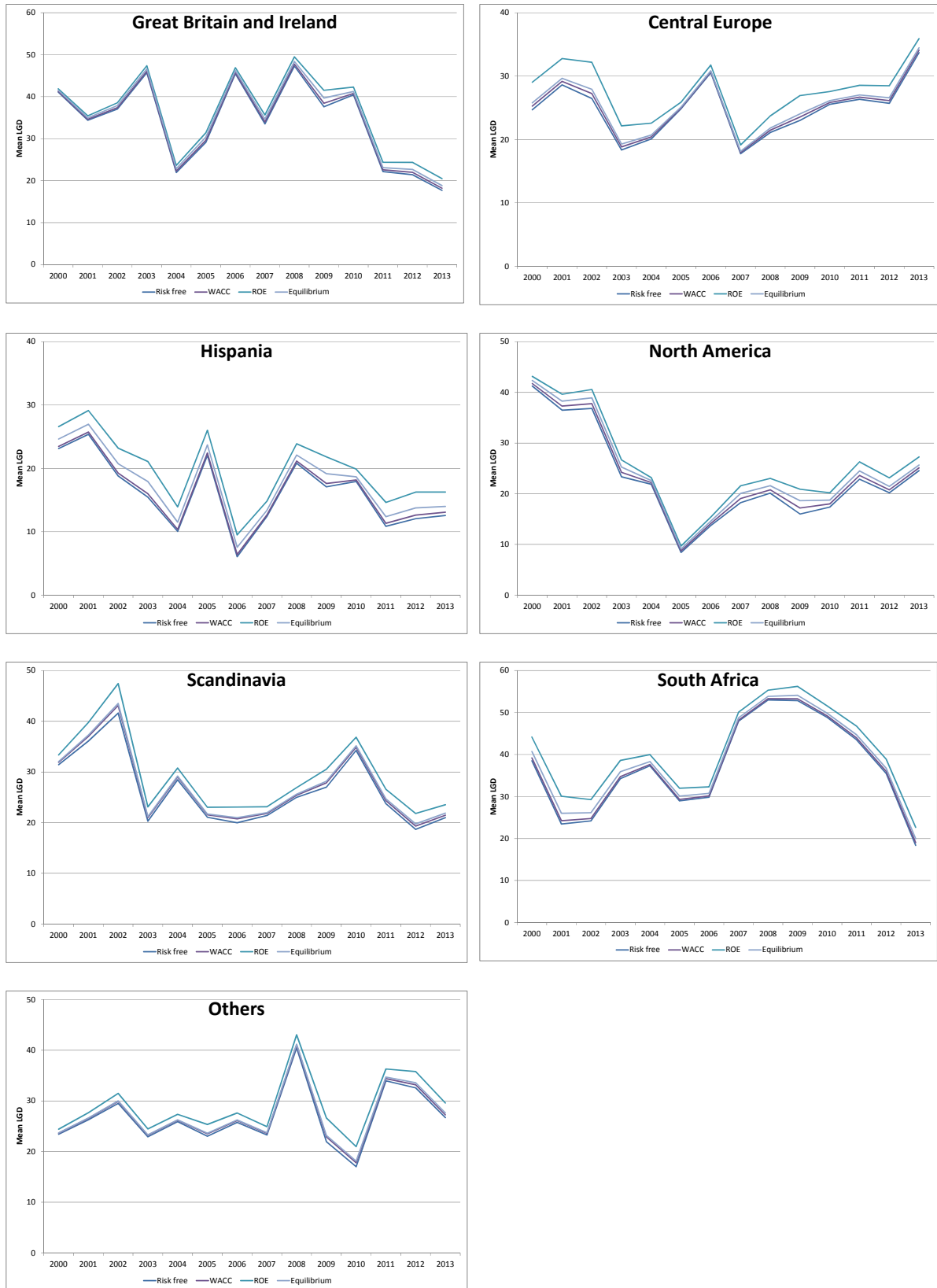


Figure 11 shows the LGDs based on the various discount rates for geographies after the correction for time to resolution bias. Furthermore, we provide the LGDs based on the various discount rates for industries after the correction for time to resolution bias in Figure 12.

5.3.3 Currency-specific risk free rates

Additionally, Figure 13 shows the LGDs based on the various discount rates for geographies, using the original cash flows and the currency-specific short term interest rate reported by the OECD as the risk free rate.

Figure 13: Mean LGD (in %), with correction for resolution time bias, by geography, local short term rate



Contract rates are excluded from this analysis as they are independent from the choice of the risk free rate. The resulting LGD ranges are similar to the ones reported in Figure 11. Minor differences are visible for South Africa (in particular during the first years) as the country has higher than average risk free rates translating into higher LGDs.

6 Outlook

The WG has a slight preference for WACC and market equilibrium returns.

- WACC is based on the loan-specific post-resolution required bank capital (Downturn LGD) and the bank-specific funding costs. The assumption is that post-default capital is a reasonable reflection of systematic risk. The approach may be of interest to smaller IRB banks as it requires fewer assumptions and models.
- Market equilibrium returns are based on the assumption that the systematic risk measure is reasonable. The realization of resolution risk is generally a non-recurring event for a borrower and the exposure to systematic risk is estimated for multiple borrowers (i.e., a risk segment) rather than on a borrower/facility level.

The risk segments may be defined by country or by industry. The risk segmentation should be based on the number of observations available after the application of filter rules and a reasonable minimum requirement may be 100 observations per risk segment and year.

To assist member banks applying these concepts, GCD may consider providing to members, on a regular basis, simple measures for systematic risk for homogenous risk segments to facilitate the appropriate reflection of systematic risk in discount rates. Members may then use these measures to determine discount rates and apply such discount rates to compute observed LGDs on the basis of the workout cash flows provided to members as part of GCD's standard template. Examples for such measures include:

- Asset correlation: this measure is capable to control for systematic information and simple estimators may be provided to GCD.
- Coefficient of variation: this measure is defined as the ratio of the standard deviation of the average LGD to the mean LGD over time and does not control for systematic information.

As the information is risk segment (and hence entity) specific as well as subject to periodic updating, GCD may consider adding an additional table to the data set that includes facility identifier (DA_LOAN_ID), reference period and measure for systematic risk. Member banks may use these informational values to apply appropriate risk premiums.

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